line in Fig. 3. In particular, this makes it possible to describe by means of function χ_1 a change for one reason or another of material element resistance to failure due to the limitedness of average stress.

The presence of function χ_2 in (5.3) makes it possible by using it to describe the effect of shear strains on the change in volume and average stress. Let deformation proceed with $\sigma = \gamma = 0$. In this case, according to (5.2) shear strains lead to a change in volume by the equation $e = \frac{2(1+\chi_2)}{1-\chi_1}\gamma'_{\alpha\beta}e'^{\alpha\beta}$. Let deformation occur without a change in volume. Then shear strains lead to a change in γ by the equation $\frac{d\gamma}{dt} = -2(1+\chi_2)\gamma'_{\alpha\beta}e'^{\alpha\beta}$ and consequently to a corresponding change in average stress. Presence of function χ_3 in (5.3) makes it possible by using it to describe the change for one reason or another of elastic shear modulus $\tilde{\mu}$.

The examples provided demonstrate that there are very extensive possibilities for describing different phenomena by introducing nondissipative inelastic strains into deformation equations for a material element.

LITERATURE CITED

1. I. I. Gol'denblat and N. A. Nikolaenko, Creep Theory for Structural Materials and Its Application [in Russian], Gostroizdat, Moscow (1960).
2. L. I. Sedov, Solid Mechanics [in Russian], Vol. 1-2, Nauka, Moscow (1970).
3. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1966).

DIRECT METHOD OF DETERMINING THE DYNAMICAL RESPONSE IN INTERACTIONS OF CONSTRUCTION ELEMENTS WITH CONCENTRATED MASSES AND RIGIDITIES

A. V. Agafonov

UDC 534.1

In many cases of calculation of construction elements (rods, plates, etc.) subjected to concentrated interactions it is sufficient to know only the strain-stress state of the considered element directly at the points of load and the element on the whole (or the whole construction) is of interest only in the sense of its integral response to the interaction.

If the concentrated interaction is given then finding this integral response is usually not difficult. In the same cases when the interaction depends on the motion of the construction element itself, determining the integral response necessitates coupling the variation of the load to the motion of the construction.

To solve such problems one uses the basic method of dynamic susceptibilities [1, 2]. According to this method the solution is constructed in two stages [1]: first, one finds separately the dynamic susceptibilities of the element and the mass (rigidity) acting on it under action of a suddenly applied concentrated force; next, one seeks the response to the interaction of the element with the mass (rigidity) from the integrodifferential equation expressing the condition of equality of the displacements of the mass and the element at the point of interaction.

At the same time there exists a possibility of developing a method which permits one to determine parameters of interest at the point of interaction bypassing the preliminary determination of the dynamic susceptibilities and shortening the process of solving the problem. Such a method can be proposed on the basis of integral transforms and the formalism of δ-functions.

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 166-172, September-October, 1987. Original article submitted June 26, 1986.

0021-8944/87/2805-0799$12.50

We will discuss the general scheme of the proposed method on the example of a one-dimensional problem. Let the behavior of a certain system in presence of concentrated masses or rigidities be described by the equation

$$L^0_{xt}(w) + L^1_t(w)\,\delta(x) = P(x, t), \tag{1}$$

where w is the unknown function (displacement); $L_{xt}{}^0(\ldots)$ is the linear operator describing the behavior of the system; $L_t{}^1(\ldots)$ is the linear operator describing the interaction of the system with a concentrated mass (rigidity); P(x, t) is the external load; x is the space coordinate; t is time.

Further, let the external load and the boundary conditions be such that it is possible to apply the Laplace transform in time and some integral transform (Fourier, Hankel, etc.) in the coordinate. Then applying integral transforms we have in the image plane

$$L_0(p,\ \nu)w_{p\nu} + L_1(\nu)w_\nu|_{x=0} = P_1(p,\ \nu). \tag{2}$$

Here $w_{p\nu}$ is the image of w; $L_0(p, \nu)$ is a polynomial determined by the operator $L_{xt}{}^0$; $L_1(\nu)$ is a polynomial determined by the operator $L_t{}^1$; $P_1(p, \nu)$ is a function determined by the load and the boundary and initial conditions, ν is the parameter of the Laplace transform; p is the parameter of the transform of the coordinate.

From the equality (2) for $w_{p\nu}$ we obtain

$$w_{p\nu} = \frac{P_1(p,\nu)}{L_0(p,\nu)} - L_1(\nu)\, w_\nu\Big|_{x=0} \frac{1}{L_0(p,\nu)}. \tag{3}$$

As we are interested only in $w|_{x=0}$, it is necessary to know the inverse transform of the expressions $P_1(p, \nu)/L_0(p, \nu)$ and $1/L_0(p, \nu)$ not for all values of x, but for x = 0, thus simplifying the solution. Let $\varphi(\nu)$ be the inverse of the expression $P_1(p, \nu)/L_0(p, \nu)$, and $\psi(\nu)$ of the expression $1/L_0(p, \nu)$ for x = 0. Then from (2) it follows that

$$w_\nu|_{x=0} = \varphi(\nu)/[1 + L_1(\nu)\psi(\nu)]. \tag{4}$$

The inversion of the image (4) can be realized either by means of tables of originals and transforms, or by means of numerical methods of inversion of Laplace transforms.

We note that the proposed method contains elements of the method of dynamical susceptibilities, but the transition to the final expression in the image plane permits one to shorten the process of obtaining the solution.

Now we consider examples of application of the method to solution of concrete problems.

1. Impact of a Mass on a Half-Infinite String

Let the half-infinite string, whose one end is fastened, be struck at distance ℓ from this end by a mass M_0 and velocity v_0. In dimensionless coordinates

$$\xi = x/l,\ \tau = \sqrt{T/m}\cdot(t/l) \tag{5}$$

the motion of the system string–mass is described (until the mass separates) by the following boundary-value problem

$$\frac{\partial^2 w}{\partial \xi^2} - \frac{\partial^2 w}{\partial \tau^2} - M\frac{\partial^2 w}{\partial \tau^2}\,\delta(\xi - 1) = 0; \tag{6}$$

$$\text{for}\quad \tau = 0\ w = 0,\ \frac{\partial w}{\partial \tau} = \begin{cases} 0 & \text{for}\quad \xi \neq 1, \\ vl & \text{for}\quad \xi = 1; \end{cases} \tag{7}$$

$$\text{for}\quad \xi = 0\ w = 0,\ \text{for}\quad \xi = \infty\ w = 0. \tag{8}$$

Here $M = M_0/m\ell$; $v = v_0/\sqrt{T/m}$; $\delta(\ldots)\delta$ is the δ-function; m is the linear mass of the string; T is the tension of the string; w is the deflection.

Applying to (6) the Laplace transform in τ and the sine Fourier transform in ξ we obtain the equation in the image plane

$$(p^2 + \nu^2)\, w_{p\nu} + \sqrt{\frac{2}{\pi}}\, M\nu^2 w_\nu\Big|_{\xi=1} \sin p = \sqrt{\frac{2}{\pi}}\, Mvl \sin p, \tag{9}$$

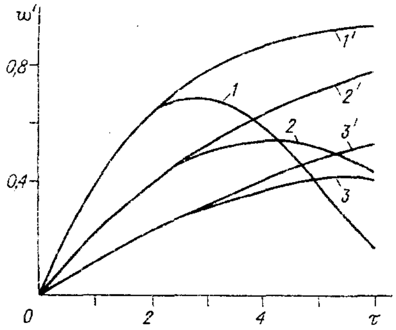

Fig. 1

where $w_{p\nu}$ is the Laplace and Fourier image of deflection (w_ν is the Laplace image); p is the parameter of the Fourier transform; ν is the parameter of the Laplace transform.

Solving (9) with respect to $w_{p\nu}$ and inverting it by Fourier, we have

$$w_\nu = M\,[vl - \nu^2 w_\nu|_{\xi=1}]\,\frac{2}{\pi}\int\limits_0^\infty \frac{\sin p\cdot\sin p\xi}{p^2+\nu^2}\,dp. \tag{10}$$

For $\xi = 1$ (the point of impact) the integral entering (10) is equal [3]

$$I = \int\limits_0^\infty \frac{\sin^2 p}{p^2+\nu^2} = \frac{\pi}{4\nu}\left(1 - e^{-2\nu}\right).$$

Taking into account this equality we get from (10)

$$w_\nu|_{\xi=1} = \frac{M}{2}vl\frac{1}{\nu}\,\frac{1-e^{-2\nu}}{1+\frac{M}{2}\nu\left(1-e^{-2\nu}\right)}. \tag{11}$$

Expanding the fraction on the right-hand side of (11) in series in powers of $e^{-2\nu}$ we find

$$w_\nu\,|_{\xi=1} = \frac{M}{2}vl\,\frac{1}{\nu\left[1+\frac{M}{2}\nu\right]}\left\{1 - \frac{e^{-2\nu}}{1+\frac{M}{2}\nu}\sum_{n=0}^{\infty}\left[\frac{\frac{M}{2}\nu e^{-2\nu}}{1+\frac{M}{2}\nu}\right]^n\right\}.$$

The factors $e^{-2n\nu}$, as usual, take into account successive reflections of waves from the fastening and the mass.

Inverting the last expression using tables of transforms and originals (see, e.g., [4]) we obtain for the deflection at the point of impact (for $\tau < 6$)

$$w|_{\xi=1} = \frac{M}{2}vl\left\{\left[1 - e^{-\frac{2\tau}{M}}\right]\sigma_0(\tau) - \left[1 - \left(1 + \frac{2[\tau-2]}{M}\right)e^{-\frac{2(\tau-2)}{M}}\right]\sigma_0(\tau-2) - \right.$$
$$\left. - \left[\frac{2(\tau-4)}{M}\right]^2 e^{-\frac{2(\tau-4)}{M}}\sigma_0(\tau-4)\ldots\right\}. \tag{12}$$

The resulting solution is valid until the moment of separation of the mass from the string. Since this separation occurs when during the reverse motion the velocity of the string at point $\xi = 1$ attains its maximal value, the moment of separation can be determined from the equation $d^2w/d\tau^2|_{\xi=1} = 0$.

The plots of the variation in time of the dimensionless deflection $w' = 2w/(Mv\ell)$ for various values of M are given in Fig. 1 where curves 1-3 are for M = 4, 8, 16; and curves 1'-3' are for an infinite string.

2. Impact of a Mass on a Beam of Finite Length

Let the beam of length 2ℓ be freely supported and be struck in the middle by a mass M_0 with velocity v_0. Then in dimensionless coordinates

$$\xi = x/l, \quad \tau = \sqrt{\frac{D}{m}}\Big/ l^2 \tag{13}$$

the motion of the beam with the mass after impact is described by the following boundary-value problem

$$\frac{\partial^4 w}{\partial \xi^4}+\frac{\partial^2 w}{\partial \tau^2}+M\frac{\partial^2 w}{\partial \tau^2}\delta(\xi-1)=0,$$
$$\text{for } \xi=0;\ 2\ w=\frac{\partial^2 w}{\partial \xi^2}=0,$$
$$\text{for } \tau=0\ w=0,\ \frac{\partial w}{\partial \tau}=\begin{cases}0 & \text{for } \xi\neq 1,\\ vl & \text{for } \xi=1.\end{cases} \tag{14}$$

In (13), (14) $M = M_0/m\ell$; $v = v_0/\sqrt{D/m\ell^2}$; m is the linear mass of the beam and D its bending rigidity.

To construct the solution we use the Laplace transform in τ and a finite sine Fourier transform in ξ. As the solution should be symmetric with respect to $\xi = 1$ the sine functions with an odd number of argument only will occur in the solution. Proceeding in this manner for the transform $w_{\nu(2k-1)}=\int\limits_0^2 w_\nu \sin\frac{(2k-1)\pi\xi}{2}d\xi$ we obtain $w_{\nu(2k-1)}\left[\left(\frac{2k-1}{2}\right)^4\pi^4+\nu^2\right]+(-1)^{k-1}\times M\nu^2 w_\nu|_{\xi=1}=(-1)^{k-1}Mvl$, and finally $w_{\nu(2k-1)}=\frac{(-1)^{k-1}M\left(vl-\nu^2 w_\nu|_{\xi=1}\right)}{\left(\frac{2k-1}{2}\right)^4\pi^4+\nu^2}$. Inverting the last expression by the Fourier transform and solving the resulting expression for $w_\nu|_{\xi=1}$ we get

$$w_\nu|_{\xi=1}=\frac{Mvl\left(\frac{2}{\pi}\right)^4\sum\limits_{k=1}^{\infty}\frac{1}{(2k-1)^4+(2\sqrt{\nu}/\pi)^4}}{1+Mv^2\left(\frac{2}{\pi}\right)^4\sum\limits_{k=1}^{\infty}\frac{1}{(2k-1)^4+(2\sqrt{\nu}/\pi)^4}}. \tag{15}$$

The sum appearing in (15) is evaluated [3]

$$S=\left(\frac{2}{\pi}\right)^4\sum_{k=1}^{\infty}\frac{1}{(2k-1)^4+(2\sqrt{\nu}/\pi)^4}=\frac{1}{2\nu\sqrt{\nu}}\left[\frac{\operatorname{sh}(2\sqrt{2\nu})+\sin(2\sqrt{2\nu})}{\operatorname{ch}(2\sqrt{2\nu})-\cos(2\sqrt{2\nu})}-\frac{1}{2}\frac{\operatorname{sh}(\sqrt{2\nu})+\sin(\sqrt{2\nu})}{\operatorname{ch}(\sqrt{2\nu})-\cos(\sqrt{2\nu})}\right].$$

Due to the complexity of the expression for S the image (15) can be inverted only numerically. At the same time the structure of the expression for S allows construction of an asymptotic analytical solution, valid for a limited time interval. Expanding (in analogy to the string) S in series in powers of $e^{-k\sqrt{2\nu}}$ and retaining the terms with the factor $e^{-2\sqrt{2\nu}}$ inclusive; we find

$$S\approx\frac{1}{2\sqrt{2}}\frac{1}{\nu\sqrt{\nu}}\left[1-e^{-\sqrt{2\nu}}\left(\sin\sqrt{2\nu}+\cos\sqrt{2\nu}\right)\right]$$

(the terms with the factor $e^{-2\sqrt{2\nu}}$ cancel).

Substituting the last expression into (15) and again expanding the right-hand side in series in powers of $e^{-k\sqrt{2\nu}}$ (up to $e^{-2\sqrt{2\nu}}$) we get

$$w_\nu|_{\xi=1}=\frac{\frac{Mvl}{2\sqrt{2}\nu}\frac{1}{\sqrt{\nu}}}{1+\frac{M}{2\sqrt{2}}\sqrt{\nu}}\left\{1-\frac{1}{1-\frac{M}{2\sqrt{2}}\sqrt{\nu}}e^{-\sqrt{2\nu}}\left[\sin\sqrt{2\nu}+\cos\sqrt{2\nu}\right]-\right.$$
$$\left.-\frac{\frac{M}{2\sqrt{2}}\sqrt{\nu}}{1+\frac{M}{2\sqrt{2}}\sqrt{\nu}}e^{-2\sqrt{2\nu}}\left[1+\sin(2\sqrt{2\nu})\right]\right\}. \tag{16}$$

To invert (16) we use the operation of convolution and tabulated formulas [4]

$$\frac{e^{-\sqrt{\alpha\nu}}}{\sqrt{\nu}}\ \dot{\to}\ \frac{1}{\sqrt{\pi\tau}}\exp\left(-\frac{\alpha}{4\tau}\right),$$
$$\frac{e^{-\sqrt{\alpha\nu}}\cos\sqrt{\alpha\nu}}{\sqrt{\nu}}\ \dot{\to}\ \frac{1}{\sqrt{\pi\tau}}\cos\left(\frac{\alpha}{2\tau}\right),$$

$$\frac{e^{-\sqrt{\alpha\nu}}\sin\sqrt{\alpha\nu}}{\sqrt{\nu}} \;\dot{\to}\; \frac{1}{\sqrt{\pi\tau}}\sin\left(\frac{\alpha}{2\tau}\right)$$

and the relationship following from the Efros theorem [2, 5]

$$\frac{F(\sqrt{\nu})}{\sqrt{\nu}} \;\dot{\to}\; \frac{1}{\sqrt{\pi t}}\int_0^\infty \exp\left(-\frac{\tau^2}{4t}\right) f(\tau)\,d\tau,$$

where $F(\nu) \;\dot{\to}\; f(\tau)$.

Inverting in (16) the rational-fraction factors with the aid of the last relationship and the factors with $e^{-\sqrt{2\nu}}$, etc., with the aid of the table relations given above and applying the convolution, we get for $w|_{\xi=1}$

$$w|_{\xi=1} = \frac{Mvl}{2\sqrt{2}}[w_1 + w_2 + w_3 + w_4]. \tag{17}$$

Here

$$w_1 = \frac{2}{\sqrt{\pi}}\sqrt{\tau} - \frac{M}{2\sqrt{2}}[1 - \varphi_1(\tau)];$$

$$w_2 = -\int_0^\tau \left[1 - \varphi_1(\tau - z) - \frac{2\sqrt{2}}{M}\varphi_2(\tau - z)\right]\frac{1}{\sqrt{\pi z}}\left(\sin\frac{1}{z} + \cos\frac{1}{z}\right)dz;$$

$$w_3 = -\frac{2\sqrt{2}}{M}\int_0^\tau \varphi_2(\tau - z)\frac{1}{\sqrt{\pi z}}\exp\left(-\frac{2}{z}\right)dz;$$

$$w_4 = -\frac{2\sqrt{2}}{M}\int_0^\tau \varphi_2(\tau - z)\frac{1}{\sqrt{\pi z}}\sin\left(\frac{4}{z}\right)dz;$$

$$\varphi_1(\tau) = \exp\left[\left(\frac{2\sqrt{2}}{M}\right)^2\tau\right]\left[1 - \Phi\left(\frac{2\sqrt{2}}{M}\sqrt{\tau}\right)\right];$$

$$\varphi_2(\tau) = \frac{2}{\sqrt{\pi}}\sqrt{\tau} - 2\frac{2\sqrt{2}}{M}\tau\varphi_1(\tau);$$

$\Phi(\tau) = \frac{2}{\sqrt{\pi}}\int_0^\tau e^{-z^2}dz$ is the probability integral.

The first term in (17) represents the deflection of an infinite beam upon impact of a mass, and the remaining terms account for the influence of the support on the deflection of the beam. As was shown by numerical calculation the main contribution to the total deflection besides the one form w_1 is due to the component w_2. The component w_4, containing under integral a fast-oscillating for $z \to 0$ function $\sin(4/z)$, and the component w_3 containing under integral a fast-decaying for $z \to 0$ function $e^{-2/z}$, do not together exceed 10% of the component w_2. The rejected components (corresponding to $k > 2$ in the expansion in $e^{-k\sqrt{2\nu}}$) will contain integrals of still faster oscillating functions of the type $\sin(6/z)$ and faster decaying functions of the type $e^{-4/z}$, etc. Therefore, one can assert that the contribution of the rejected terms will be yet smaller than the contribution of w_3 and w_4 and the proposed asymptotic solution can be used at least for $\tau \le 6$.

The plots of the variation of the dimensionless deflection $w_1' = 2\sqrt{2}w/Mv\ell$ for various values of M are shown in Fig. 2, where curves 1-3 show the deflection for M = 2, 4, 6; curves 1'-3' show the same for an infinite beam.

3. Action of a Concentrated Force on an Infinite String Fastened to a Membrane.

Let us consider a string with the transverse cross-sectional area F, under tension T, fastened to a membrane of thickness h under tension T_1. The string is acted upon by a concentrated force $P_0 f(t)$. The material of the string and membrane is assumed to be the same for simplicity, and the tensions such that the stresses in the string and the membrane are identical, i.e., $T_1/h = T/F$. In dimensionless coordinates $\xi = \frac{x}{h}$, $\eta = \frac{y}{h}$, $\tau = \frac{\sqrt{T_1/\rho h}}{h}t$ the motion of such a system is described by the equation (the axis ξ is directed along the string)

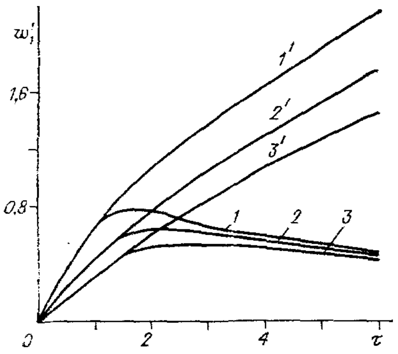

Fig. 2

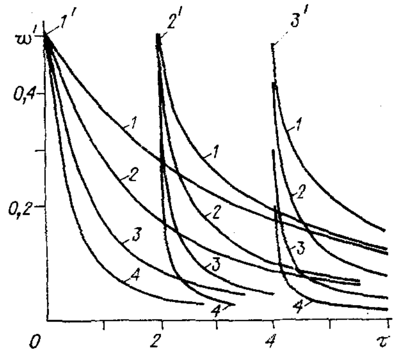

Fig. 3

$$\frac{\partial^2 w}{\partial \xi^2} - \frac{\partial^2 w}{\partial \eta^2} - \frac{\partial^2 w}{\partial \tau^2} + a^2 \left[\frac{\partial^2 w}{\partial \xi^2} - \frac{\partial^2 w}{\partial \tau^2}\right] \delta(\eta) + \frac{P_0}{T_1} f(\tau)\,\delta(\xi)\,\delta(\eta) = 0, \; a^2 = \frac{T}{T_1 h} = \frac{F}{h^2}. \tag{18}$$

The initial and boundary conditions (for $\xi = \pm\infty$, $\eta = \pm\infty$) are zero.

Applying to (18) the double Fourier transform in ξ and η and the Laplace transform in τ, we obtain for the image $w_{pq\nu}$ the equality $(p^2 + q^2 + \nu^2)\, w_{pq\nu} + \frac{1}{\sqrt{2\pi}} a^2 (p^2 + \nu^2)\, w_{p\nu}|_{\eta=0} = \frac{1}{2\pi} \frac{P_0}{T_1} f_\nu$ [f_ν is the image of $f(\tau)$], from which we get $w_{pq\nu} = \frac{1}{\sqrt{2\pi}} \left[\frac{1}{\sqrt{2\pi}} \frac{P_0}{T_1} f_\nu - a^2 (p^2 + \nu^2)\, w_{p\nu}|_{\eta=0}\right] \frac{1}{p^2 + q^2 + \nu^2}$. Inverting this last expression with respect to q using the known value of the integral [3]

$$I = \int_0^\infty \frac{dq}{q^2 + a^2} = \frac{\pi}{2} \frac{1}{a}$$

and solving the resulting equality for $w_{p\nu}|_{\eta=0}$ we have

$$w_{p\nu}|_{\eta=0} = \frac{P_0}{2T_1} \frac{1}{\sqrt{2\pi}} \frac{f_\nu}{\sqrt{p^2+\nu^2}\left[1 + \frac{a^2}{2}\sqrt{p^2+\nu^2}\right]}. \tag{19}$$

To invert the image (19) we use the relationship following from the Efros theorem

$$\frac{F\left(\sqrt{\nu^2+p^2}\right)}{\sqrt{\nu^2+p^2}} \doteqdot \int_0^\tau J_0\left(p\sqrt{\tau^2 - z^2}\right) f(z)\, dz,$$

where $F(\nu) \doteqdot f(\tau)$; $J_0(\ldots)$ is the Bessel function of the first kind of index 0. By inverting (19) by Fourier and Laplace transforms using the last relationship, we find for $f_\nu = 1$ ($f(\tau) = \delta(\tau)$)

$$w(\xi, \tau)|_{\eta=0} = \frac{P_0 h}{T} \frac{1}{\pi} \int_0^\tau e^{-\frac{2}{a^2} z} \int_0^\infty J_0\left(p\sqrt{\tau^2 - z^2}\right) \cos p\xi\, dp.$$

Since [3, 6]

$$\int_0^\infty J_0\left(p\sqrt{\tau^2 - z^2}\right) \cos p\xi\, dp = \begin{cases} \dfrac{1}{\sqrt{\tau^2 - \xi^2 - z^2}}, & z^2 < \tau^2 - \xi^2, \\ 0, & z^2 > \tau^2 - \xi^2, \end{cases}$$

then

$$w(\xi, \tau)|_{\eta=0} = \frac{P_0 h}{T} \frac{1}{\pi} \int_0^{\sqrt{\tau^2-\xi^2}} \frac{e^{-\frac{2}{a^2} z}}{\sqrt{\tau^2 - \xi^2 - z^2}}\, dz = \frac{P_0 h}{T} \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{2}{a^2}\sqrt{\tau^2-\xi^2}\sin\varphi}\, d\varphi$$

$$(z = \sqrt{\tau^2 - \xi^2}\, y, \; y = \sin\varphi).$$

The plots of variation in time of the dimensionless deflection $w' = \frac{T}{P_0 h} w(\xi, \tau)|_{\eta=0}$ for different values of ξ and a^2 and are shown in Fig. 3, where curves 1'-3' are for ξ = 0, 2, 4; and curves 1-4 for $a^2 = 4, 2, 1, 0.5$. It is apparent that for fixed value of the ratio T/h with decreasing parameter a^2 the drop of deflection in time becomes steeper, while for $a^2 \to \infty$ the character of the time variation of the deflection approaches the variation of deflection for an isolated string.

LITERATURE CITED

1. T. D. Kudrina and V. I. Mamai, in: Statics and Dynamics of Thin-Walled Constructions [in Russian], Izd. MGU, Moscow (1980).
2. L. I. Slepyan, Nonstationary Elastic Waves [in Russian], Sudostroenie, Leningrad (1972).
3. A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, Integrals and Series. Elementary Functions [in Russian], Nauka, Moscow (1981).
4. G. Doetsch, Practical Guide to Application of Laplace Transform [Russian translation], GIFML, Moscow (1960).
5. V. A. Ditkin and A. P. Prudnikov, Handbook of Operational Calculus [in Russian], Vysshaya Shkola, Moscow (1965).
6. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, New York (1965).